Robustness Study of Generic and Non-Generic 3R Positioning Manipulators

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Abstract— This paper presents a robustness study of 3R manipulators and aims at answering the following question: are generic manipulators more robust than their non-generic counterparts? We exploit several properties specific to 3R manipulators such as singularities, cuspidality, homotopy classes, and path feasibility, in order to find some correlations between genericity and robustness concepts. It turns out that the farther a manipulator is from non-generic ones, the more robust it is with respect to homotopy class and number of cusps. Besides, we state that the proximity of a manipulator to non-generic frontiers may severely affect its robustness with respect to path-feasibility. Finally, we notice that the dexterity and the accuracy of 3R manipulators do not depend on genericity.

Keywords—Robust design, genericity, path-feasibility, singularity, cuspidality, homotopy class, serial manipulator.

I. INTRODUCTION

The performance functions of a manipulator are numerous. Its dexterity and accuracy, the shape and the size of its workspace are some criteria that can be used during its design stage.

This paper deals with a robustness study of 3R manipulators, which are serial manipulators and composed of three actuated revolute joints. A detailed analysis of 3R manipulator singularities is presented in [1] and the notion of genericity is introduced in [2]. A manipulator is generic if its singularities are generic (they do not intersect in the joint space). Non-generic manipulators form hyper-surfaces dividing the space of manipulators into different sets of generic ones. Consequently, most manipulators are generic.

The concept of robust design was introduced by G.Taguchi. He proposed the concept of parameter design to improve the quality of a product whose manufacturing process involves significant variability and noise [3]. As a matter of fact, robust design aims at minimizing the sensitivity of performances to variations without controlling the causes of these variations that can arise from a variety of sources, including manufacturing operations, variations in material properties, and the operating environment [4].

Here, the main issue is to know whether generic manipulators are more robust than non-generic manipulators or not. This issue is critical because the majority of industrial robots are supposed to be non-generic. In fact, they are usually non-generic due to the simplification of their geometric parameters. On the one hand, some authors [1, 5] claimed that non-generic manipulators should be less robust than their generic counterparts. On the other hand, assuming that the lower the complexity of a design, the higher its robustness, we can expect the opposite.

First, some properties specific to 3R manipulators and useful for the comprehension of the study are presented. The geometry of a 3R manipulator is described and some notions such as singularities, cuspidality, genericity, homotopy classes, and path feasibility are mentioned. Then, robustness of generic and non-generic manipulators are compared with respect to their homotopy class and to the path-feasibility. Finally, the influence of the genericity (non-genericity) of a manipulator on its dexterity and accuracy is analyzed.

II. Preliminaries

A. Geometry of 3R Manipulators

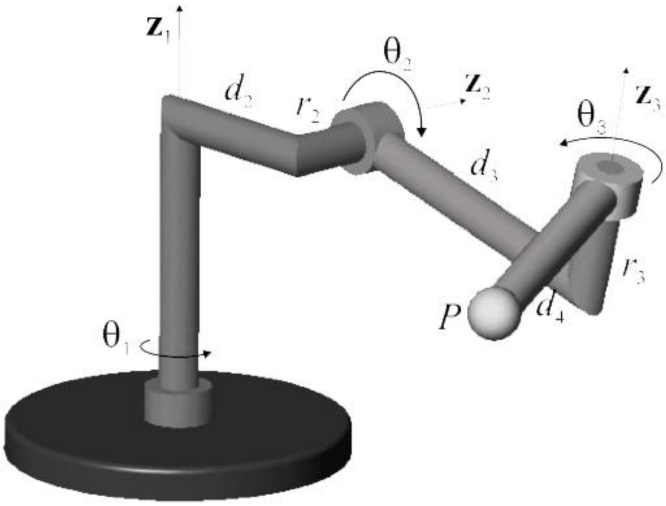


Figure 1 : An Orthogonal 3R Manipulator

Figure 1 depicts an orthogonal 3R manipulator. It is a serial manipulator and is made up of three actuated revolute joints. Modified D-H parameters [6] are used:

- d_2, d_3, d_4, r_2, r_3, $\alpha_2 = \angle(\mathbf{z}_1, \mathbf{z}_2)$, and $\alpha_3 = \angle(\mathbf{z}_2, \mathbf{z}_3)$ are the geometric parameters of the manipulator;
- θ_1, θ_2, and θ_3 are the actuated joint angles.

This manipulator is orthogonal because $\alpha_2 = -90°$ and $\alpha_3 = 90°$. Most industrial robots are composed of a positioning structure and a wrist. Usually, the positioning structure is a 3R manipulator and the wrist is composed of three revolute joints with convergent axes. For example, the positioning structure of PUMA robots is a 3R manipulator, with geometric parameters kinematically equivalent to: $d_2 = 0$, $r_2 = 0$, $r_3 = 0$, $\alpha_2 = 90°$, and $\alpha_2 = 180°$.

B. Singularities

Serial 3R positioning manipulators presented here have only positioning singularities (referred to as "singularity" in the rest of the paper). A singularity can be characterized by a set of joint configurations that nullifies the determinant of the Jacobian matrix. They divide the joint space into at least two domains called aspects [7].

The aspects are the maximal free-singularity domains in the joint space. Burdick [1] defines the critical point surfaces as the connected and continuous subset of singularities. Their corresponding images in the workspace are defined as critical value surfaces. The critical value surfaces divide the workspace into different regions with different number of inverse kinematic solutions or postures [8].

For a 3R manipulator, the joint space has the structure of a 3-dimensional torus. The singularities can be studied on the 2-dimensional (θ_2, θ_3)-torus because they do not depend on θ_1.

C. *Cuspidal Manipulators*

A cuspidal manipulator can change posture without meeting any singularity. The existence of such manipulators was discovered simultaneously in [9] and [10]. In [11], a theory and methodology were introduced to characterize new uniqueness domains in the joint space of cuspidal manipulators. The only possible region of the workspace where a cuspidal manipulator can change posture without meeting singularity is a region with four inverse kinematic solutions. Characterization of cuspidal manipulators is difficult. Obviously, observation of several examples of manipulators gave rise to some conjectures by authors. In fact, some of them state that manipulators with simplifying geometric conditions like intersecting, orthogonal or parallel joint axes cannot avoid singularities when changing posture [8, 12]. Others claim that manipulators with arbitrary kinematic parameters are cuspidal [1, 9]. Neither the first, nor the second idea can be stated in a general way. In [13], a new characterization of cuspidal manipulators was done: a 3-DOF positioning manipulator can change posture without meeting a singularity if and only if there exists at least one point in its workspace with exactly three coincident inverse kinematic solutions and such a point is called a cusp point.

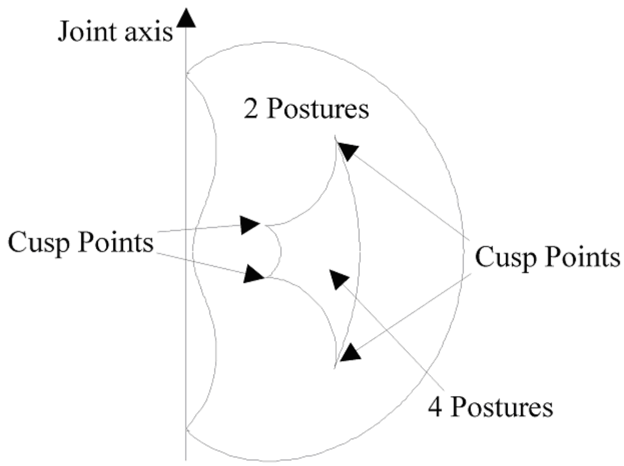


Figure 2: Cusp Points in the Workspace Section of a Cuspidal Manipulator

Figure 2 shows the critical value surfaces for a cuspidal manipulator, $d_2 = 1$, $d_3 = 2$, $d_4 = 1.5$, $r_2 = 1$, $r_3 = 0$, $\alpha_2 = -90°$ and $\alpha_3 = 90°$, in a cross-section of its workspace. There are four cusp points and two regions with four and two possible postures, respectively. Numerical and graphical methods are used to check the conditions of existence of a cusp point. Consequently, it provides a useful tool for the purpose of manipulator design. In general, it is not possible to write the conditions of existence of cusp points in an explicit expression of the DH-parameters [5].
However, for a family of 3R manipulators with orthogonal axes, Baili et al. [14] found an explicit condition of the existence of cusp points, which depends only on DH-parameters.

D. Generic Manipulators

According toBurdick [1], a generic manipulator has to respect the two following conditions:

- its Jacobian matrix has rank 2 at all the critical points;
- all singular points, θ_S, must satisfy the following condition:

$$\frac{\partial[\det(\mathbf{J}(\theta_s))]}{\partial\theta_i} \neq 0 \quad \text{for } i \text{ equal to 1 or/and 2}$$

Pai [2] claimed that a generic manipulator is defined as one having no intersection of its smooth singularity surfaces in the joint space, and showed that the two foregoing conditions are equivalent for a 3R manipulator.
Simplifications in manipulator geometry, like intersecting or parallel joint axes, often lead to non-genericity. In fact, a major part of industrial manipulators are non-generic. However, many non-generic manipulators have complicated DH-parameters [1, 12]. Besides, generic manipulators have usually stable global kinematic properties under small changes in their design parameters.

E. Homotopy Classes

Homotopy classes were defined in [15] only for generic, quaternary manipulators. A quaternary manipulator is defined as one having four inverse kinematic solutions. A binary manipulator has only two solutions. Two quaternary generic manipulators are homotopic if the singularity surfaces of one manipulator can be smoothly deformed to the singularity surfaces of the other. Burdick [1] showed that two homotopic manipulators have the same multiplicity of their kinematic maps. So, homotopic manipulators have the same maximum number of inverse kinematic solutions per aspect. Therefore, all the manipulators homotopic to a cuspidal (resp. non-cuspidal) manipulator are cuspidal (resp. non-cuspidal).

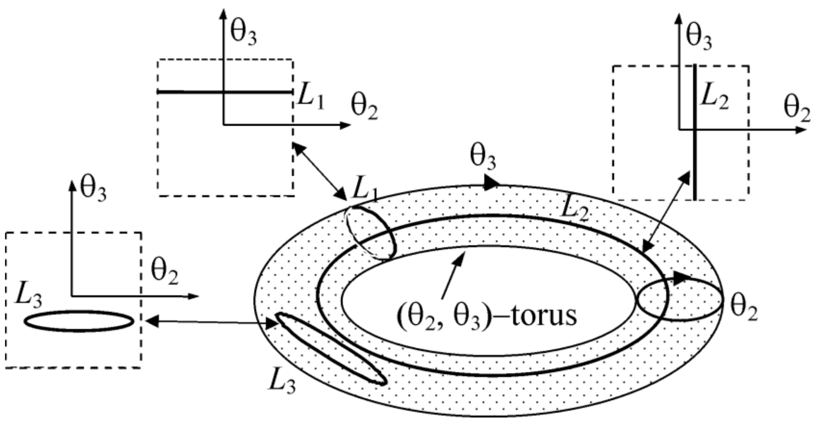


Figure 3: Some Loops of Homotopy Classes on the Torus

A singularity surface forms a loop when projected onto the surface of (θ_2, θ_3)-torus. Therefore, there are as many homotopy classes as ways of encircling two generators of torus. Figure 3 shows three different homotopy classes. Lines L_1 and L_2 plotted in the square

($- \pi \leq \theta_2 \leq \pi$, $- \pi \leq \theta_3 \leq \pi$) tally with the circles plotted along θ_2-generator and θ_3-generator of the torus, respectively. On the contrary, L_3 does not encircle any of the two torus generators. Thus, L_3 is homotopic to a point.

The homotopy class of a singularity surface can be defined by a set of two integers (n_2, n_3). Integer n_2 (resp. n_3) characterizes the number of times the loop associated with the singularity surface encircles the θ_2-generator (resp. θ_3-generator) of (θ_2, θ_3)-torus. Accordingly, the homotopy class of a generic manipulator is characterized by a series of couples (n_2, n_3), which define the homotopy classes of each of its singular surfaces.

The way to determine the homotopy class of a given generic manipulator is to track each branch, and to count for the number of "jumps" between two opposite sides of the square representation. At each jump, n_2 and n_3 are either increased or decreased, according to whether the jump occurs from $-\pi$ to π or from π to $-\pi$, respectively.

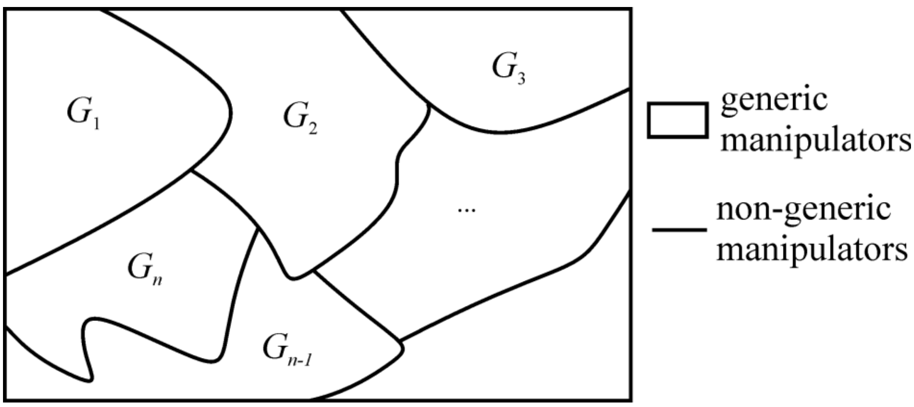


Figure 4: Frontiers of Generic Manipulators in the Geometric Parameters Space

For example, the homotopy class of L_1 (resp. L_2, L_3) is (1,0), (resp. (0,1), (0,0)). The number and the homotopy class of the singularity surfaces define a set of homotopic generic manipulators. The set of all the 3R positioning manipulators is divided into subsets of homotopic generic manipulators split by subsets of non-generic manipulators [15], as shown in Fig.4.

F. Path-feasibility

In many cases, such as in welding tasks, the end-effector has to follow a path in the workspace. For a non-cuspidal manipulator, a path is feasible if it can be followed in one single aspect, i.e., without meeting singularities or joint limits. The images of the aspects in the workspace define the regions of feasible paths [16].

III. ROBUSTNESS STUDY OF 3R MANIPULATORS

To the best of our knowledge, there is no thorough study on robustness of generic and non-generic manipulators in the literature. As mentioned before, generic manipulators have stable global kinematic properties under small changes in their design parameters. However, the question remains, is it enough to claim that generic manipulators are more robust than non-generic manipulators?

In order to answer this question, we study the robustness of 3R manipulators with respect to their homotopy class. Then, we focus on their robustness with respect to path feasibility. Finally, we study the sensitivity of the pose of their end-effector to variations in their geometric parameters.

A. Robustness with respect to Homotopy Classes

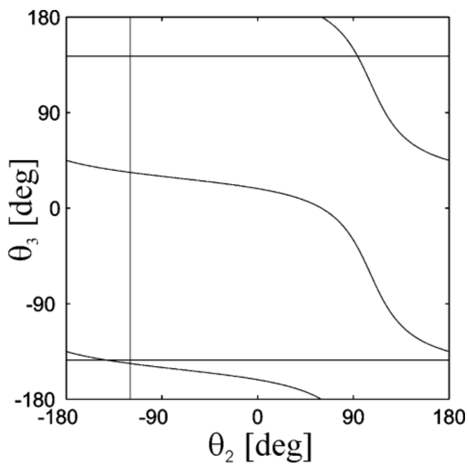


Figure 5: Joint Space of the Non-Generic 3R Manipulator defined by: $d_2 = 1$, $d_3 = 2$, $d_4 = 2.5$, $r_2 = 1$, $r_3 = 0$, $\alpha_2 = -60°$, $\alpha_3 = 90°$

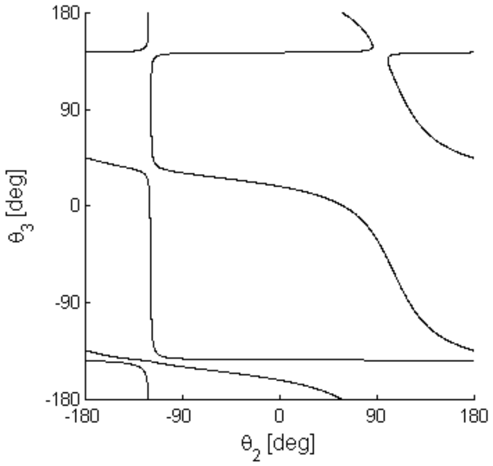


Figure 6: Joint Space of the Generic Manipulator, Class 2(1,1): $d_2 = 1$, $d_3 = 2$, $d_4 = 2.5$, $r_2 = 1$, $r_3 = 0.01$, $\alpha_2 = -59°$, $\alpha_3 = 90°$

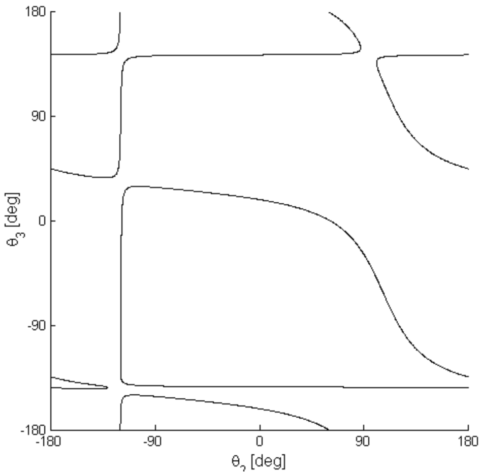


Figure 7: Joint Space of the Generic Manipulator, Class 2(0,0): $d_2 = 1$, $d_3 = 2$, $d_4 = 2.5$, $r_2 = 1$, $r_3 = 0.01$, $\alpha_2 = -61°$, $\alpha_3 = 90°$

Figures 5, 6, and 7 depict the joint space of a non-generic manipulator and those of two generic manipulators close to the non-generic manipulator, respectively. Indeed, only r_3 and α_2 change from one manipulator to the other and they vary a little.

The homotopy class of the first generic manipulator is 2(1,1) because its two singularity surfaces encircle θ_2-generator and θ_3-generator. However, the homotopy class of the second generic manipulator is 2(0,0) due to the fact that its joint space includes only one singularity surface, which encircles neither the θ_2-generator, nor the θ_3-generator (this can be more easily seen by "gluing" the opposite sides of the square).

We can conclude from this example that a non-generic manipulator faced with small geometric variations becomes a generic manipulator, of which the homotopy class is either 2(1,1) or 2(0,0). The topology of the singularity surfaces of a manipulator depends on its homotopy class. Therefore, non-generic manipulators and their adjoined generic-manipulators are not robust with respect to homotopy classes and the topology of the singularity surfaces.

According to section II-E, all the manipulators homotopic to a cuspidal (resp. non-cuspidal) manipulator are cuspidal (resp. non-cuspidal). Therefore, a manipulator that is supposed to be cuspidal can become non-cuspidal when faced with geometric variations. Such a manipulator not necessarily will be able to execute a non singular change of posture.

B. Robustness with respect to Path Feasibility

First, we introduce the definition of the robustness of a manipulator with respect to Path Feasibility.

<u>Definition:</u> Robustness of a Manipulator with respect to Path Feasibility

A manipulator is robust with respect to Path Feasibility if all the paths feasible with its nominal geometric parameters are still feasible with its real geometric parameters, i.e., when faced to geometric variations.

Here, we compare some pairs of manipulators in order to study the influence of the genericity and the non-genericity of a manipulator on the path feasibility.

1) First example

Let us consider the 3R manipulator with nominal geometric parameters: $d_2 = 1$, $d_3 = 0.6$, $d_4 = 2$, $r_2 = 1$, $r_3 = 0.1$, $\alpha_2 = -90°$, and $\alpha_3 = 90°$.

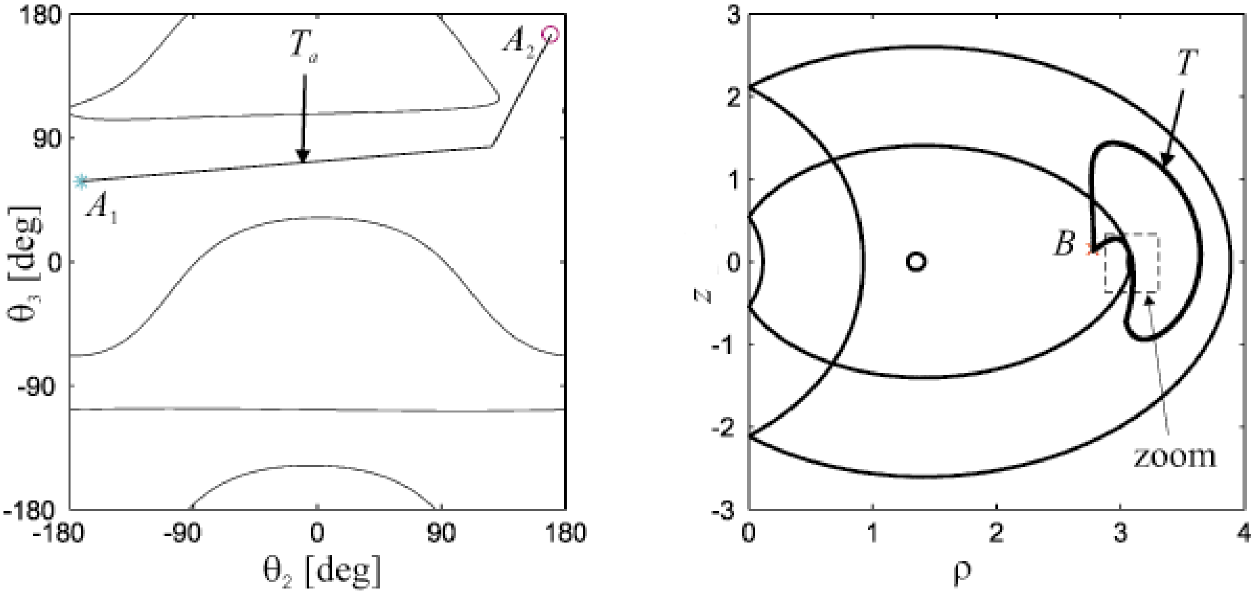


Figure 8: *T* is Path Feasible

Figure 8 depicts its joint space and workspace. By following T_a from point A_1 to point A_2 in the joint space, the end-effector *P* of the manipulator follows a closed path *T* from *B* to *B* with a change of posture. Indeed, points A_1 and A_2 of the joint space are two distinct pre-images of *B* corresponding to two different postures of the manipulator.

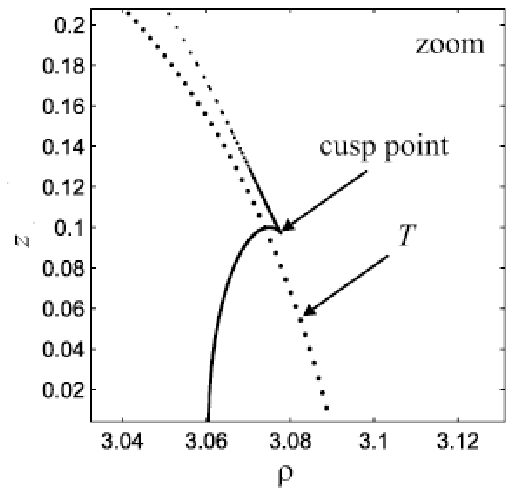


Figure 9: Zoom on T around the Cusp Point

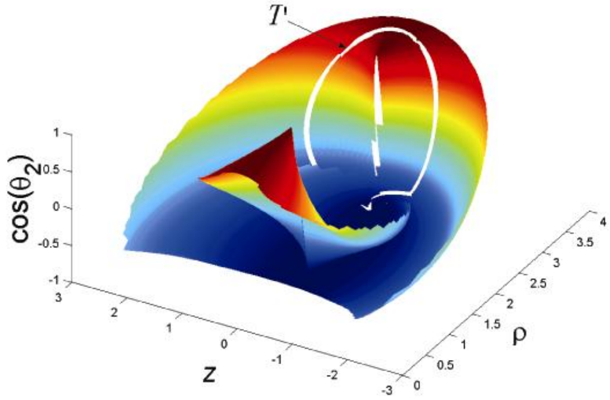


Figure 10: T lies in a Region of Feasible Paths

As mentioned in section II-C, only cuspidal manipulators can change posture without meeting any singularity. According to Fig.9, path T passes near a cusp point. Fig. 10 depicts the region of feasible paths, which includes T.

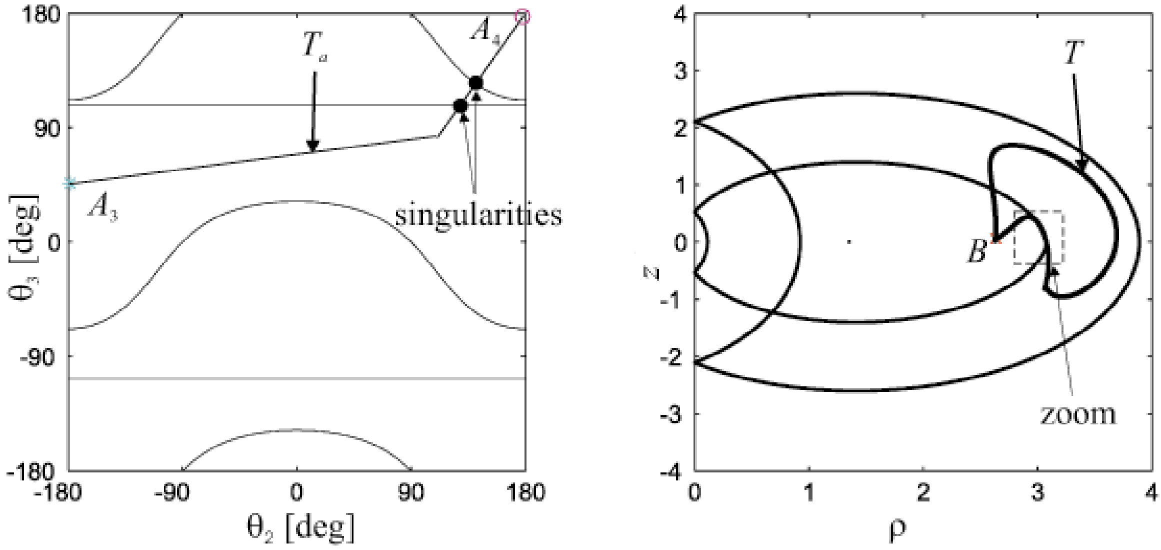


Figure 11: T is not Path Feasible

Let us assume that r_3 changed a little and becomes zero. Figure 11 depicts the joint space and the workspace of the new manipulator.

In this case, we can notice that the manipulator cannot follow T without meeting any singularity. Indeed, T_a, the image of T under the inverse geometric operator of the manipulator, cuts a singularity surface twice in the joint space.

Figure 12 depicts a zoom in T around the corresponding singular points in the workspace.

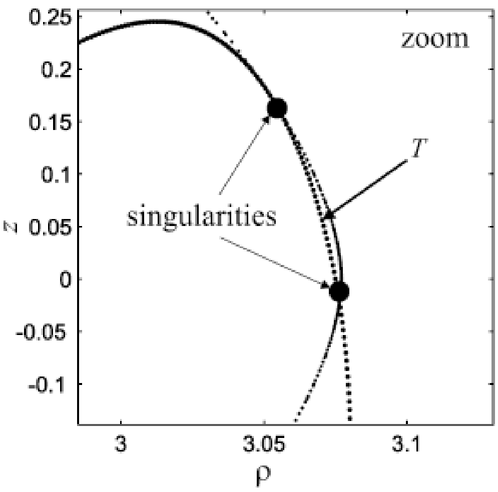


Figure 12: Zoom on T and Singular Points

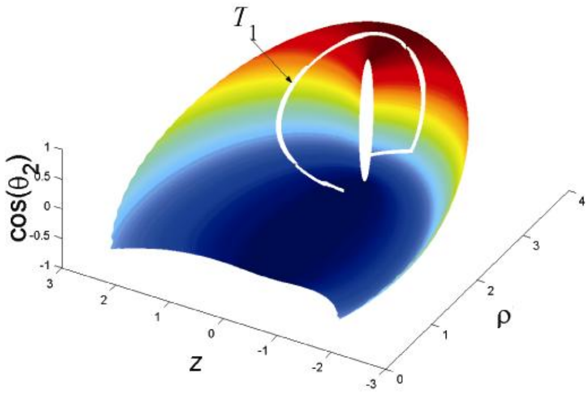


Figure 13: T_1 lies in a Region of Feasible Paths

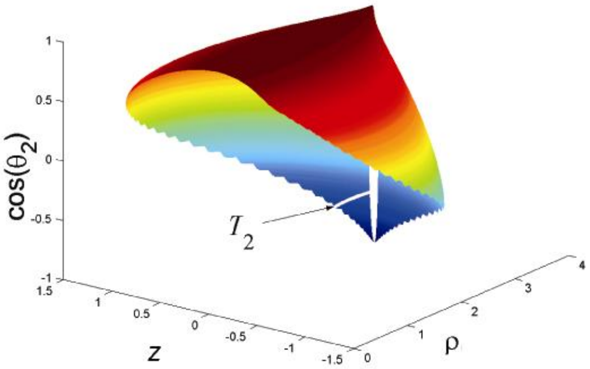


Figure 14: T_2 lies in a Region of Feasible Paths

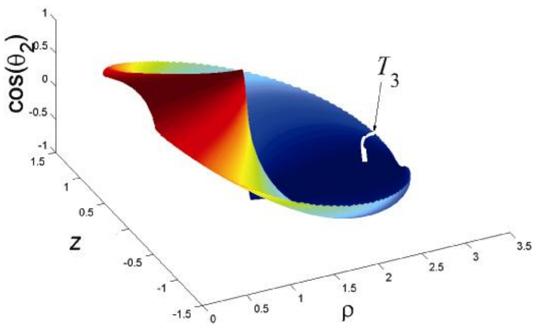


Figure 15: T_3 lies in a Region of Feasible Paths

As a matter of fact, T is the union of T_1, T_2, and T_3, which are all feasible. Their corresponding regions of feasible paths are depicted by Figs. 13, 14, and 15, respectively.

Furthermore, we can notice that the two previous manipulators are generic but do not belong to the same homotopy class. Indeed, the homotopy class of the first manipulator is 2(1,0) + 1(0,0) whereas the one of the second manipulator is 4(1,0). Accordingly, a small variation in geometric parameter r_3 can change substantially the topology of the singularity surfaces of the manipulator. Likewise, the number of cusp points changes because the first manipulator has two cusp points whereas the second one is not cuspidal. Therefore, these two manipulators do not belong to the same set of generic manipulators but ‘‘adjoin’’ the same non-generic manipulator.

2) *Second example*

Let us compare the path feasibility of T by means of two generic manipulators far enough from the previous non-generic manipulator in the set of geometric parameters.

Note: A manipulator is supposed to be far enough from another manipulator in the set of geometric parameters if these two manipulators cannot become the same in presence of given variations in their geometric parameters.

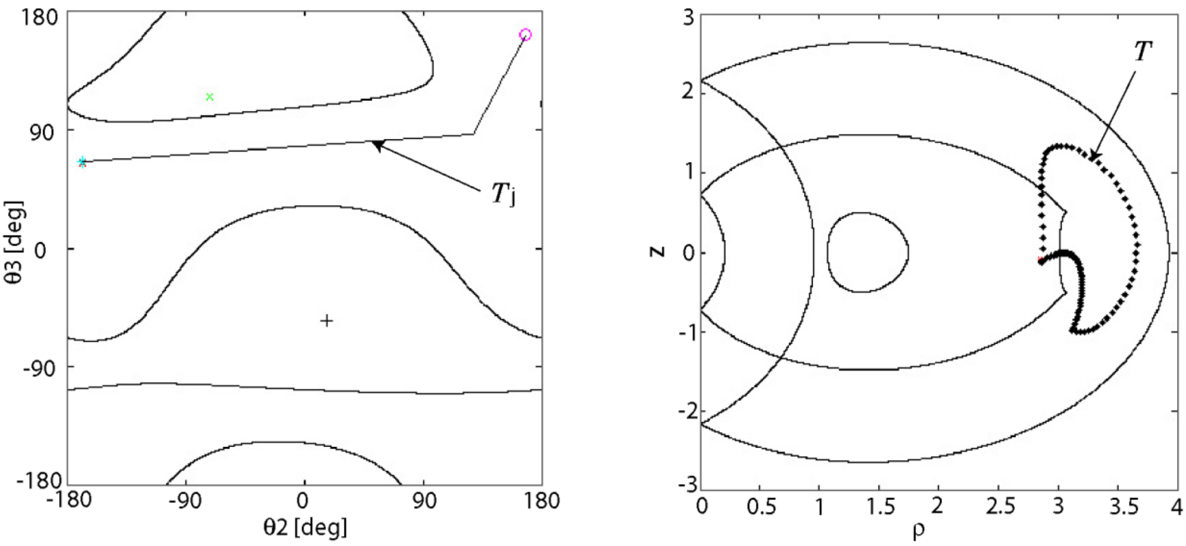


Figure 16: Manipulator d_2 = 1, d_3 = 0.6, d_4 = 2, r_2 = 1, $\boldsymbol{r_3}$ **= 0.5**, α_2 = -90°, and α_3 = 90°: T is Path Feasible

Figure 16 depicts the joint space and the workspace of the 3R manipulator of which the geometric parameters are d_2 = 1, d_3 = 0.6, d_4 = 2, r_2 = 1, $\boldsymbol{r_3}$ **= 0.5**, α_2 = -90°, and α_3 = 90°. Tj is the image of path T into the joint space of the manipulator and does not meet any singularity branch. Consequently, T is path feasible.

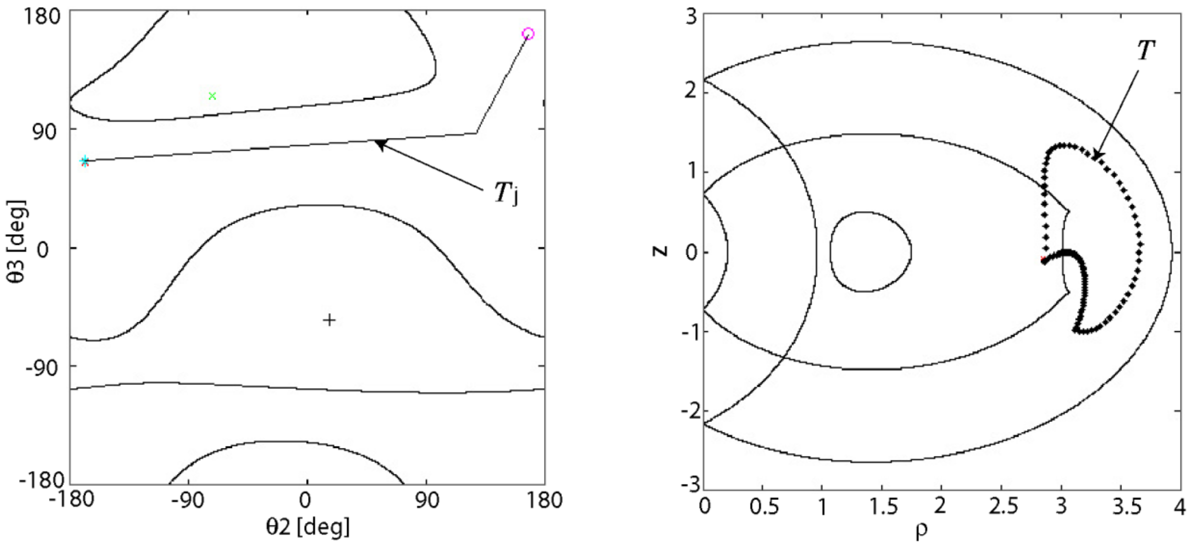


Figure 17: Manipulator $d_2 = 1$, $d_3 = 0.6$, $d_4 = 2$, $r_2 = 1$, $\boldsymbol{r_3 = 0.4}$, $\alpha_2 = -90°$, and $\alpha_3 = 90°$: T is still Path Feasible

Figure 17 depicts the joint space and the workspace of the 3R manipulator of which the geometric parameters are $d_2 = 1$, $d_3 = 0.6$, $d_4 = 2$, $r_2 = 1$, $\boldsymbol{r_3 = 0.4}$, $\alpha_2 = -90°$, and $\alpha_3 = 90°$. Likewise, *T*j is the image of path *T* into the joint space of the manipulator and does not meet any singularity branch. It means that *T* is still path feasible.

3) Third example

Let us consider the 3R manipulator defined with the following nominal parameters are $d_2 = 0$, $d_3 = 2$, $d_4 = 1.5$, $r_2 = 1$, $r_3 = 0$, $\alpha_2 = -90°$, and $\alpha_3 = 90°$.

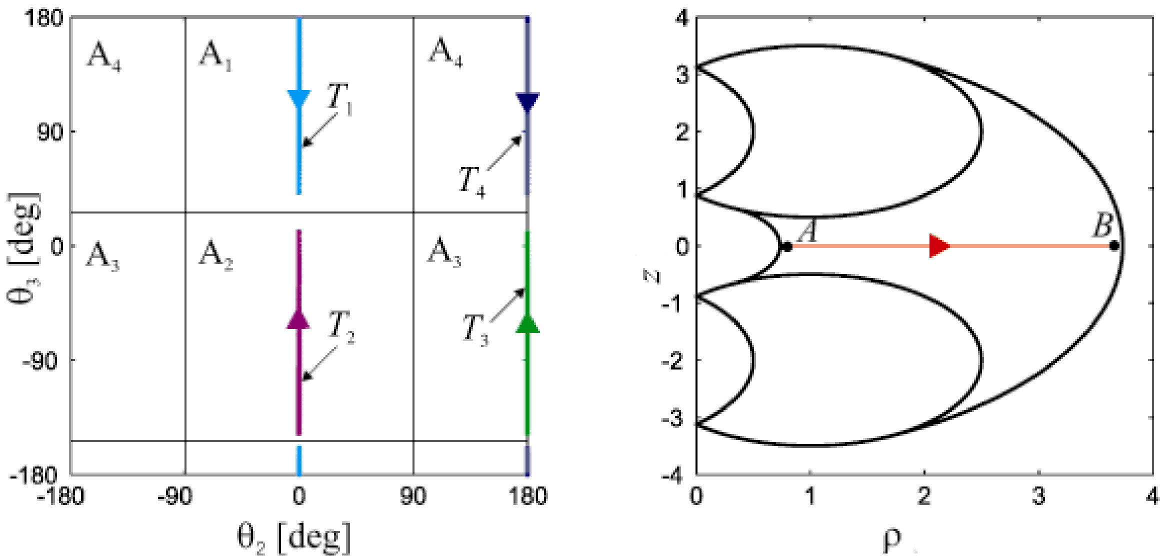


Figure 18: *AB* is Path Feasible

Figure 18 depicts its joint space and workspace. This manipulator is non-generic because its singularity surfaces intersect in the joint space.

Here, the manipulator must follow path *AB*. The coordinates of *A* and *B* in the workspace (ρ, z) are (0.76,0) and (3.7,0), respectively. According to Fig 18, the joint space of the manipulator is composed of four aspects: A_1, A_2, A_3 and A_4, i.e.: four areas free of singularity. Moreover, T_1, T_2, T_3, and T_4 are the pre-images of path *AB*. They are included in A_1, A_2, A_3, and A_4, respectively.

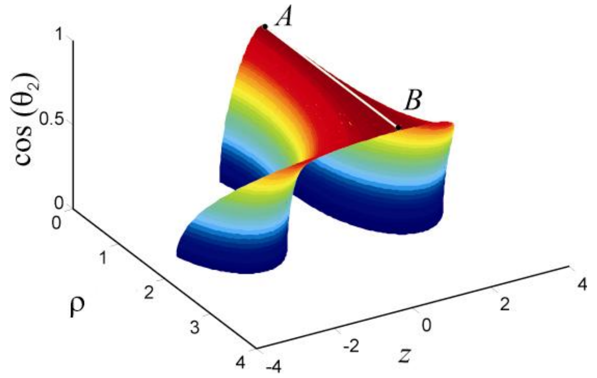


Figure 19: Image of aspect A_1: Region 1 of Feasible Paths

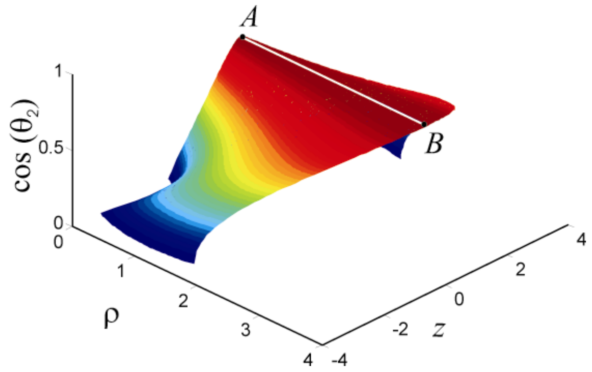


Figure 20: Image of aspect A_2: Region 2 of Feasible Paths

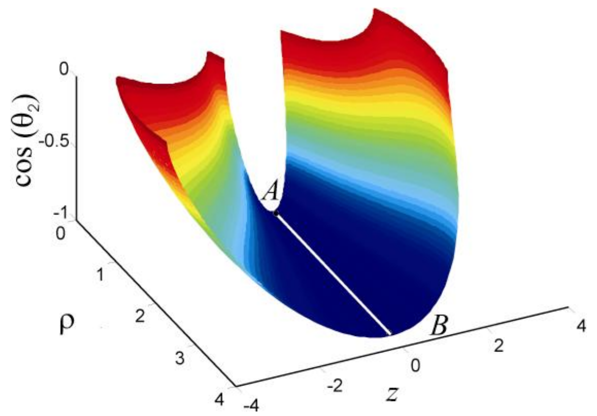


Figure 21: Image of aspect A_3: Region 3 of Feasible Paths

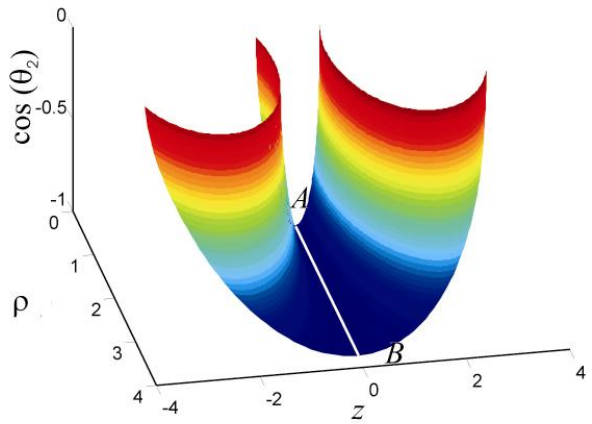


Figure 22: Image of aspect A_4: Region 4 of Feasible Paths

Figures 19, 20, 21, and 22 depict the regions of feasible paths of the manipulator, i.e.: the images of aspects A_1, A_2, A_3 and A_4 under the geometric operator of the manipulator.

The horizontal plane of these figures depicts the workspace of the manipulator, defined by ρ ($\rho = (x^2+y^2)^{1/2}$) and z coordinates. The vertical axis, which is the cosine of joint angle θ_2, is used to distinguish the regions of feasible paths in the workspace of the manipulator. This representation is particularly interesting to visualize the regions of feasible paths of cuspidal manipulators, [16].

We can notice that path *AB* lies in all the regions of feasible paths. It means that *AB* is feasible with this non-generic manipulator. However, is *AB* still feasible when geometric parameters of the manipulator vary a bit?

Let us consider the 3*R* manipulator with geometric parameters: $d_2 = 0.1$, $d_3 = 2$, $d_4 = 1.5$, $r_2 = 1$, $r_3 = 0$, $\alpha_2 = -90°$, and $\alpha_3 = 90°$. This manipulator differs from the previous one by d_2, which is slightly perturbed.

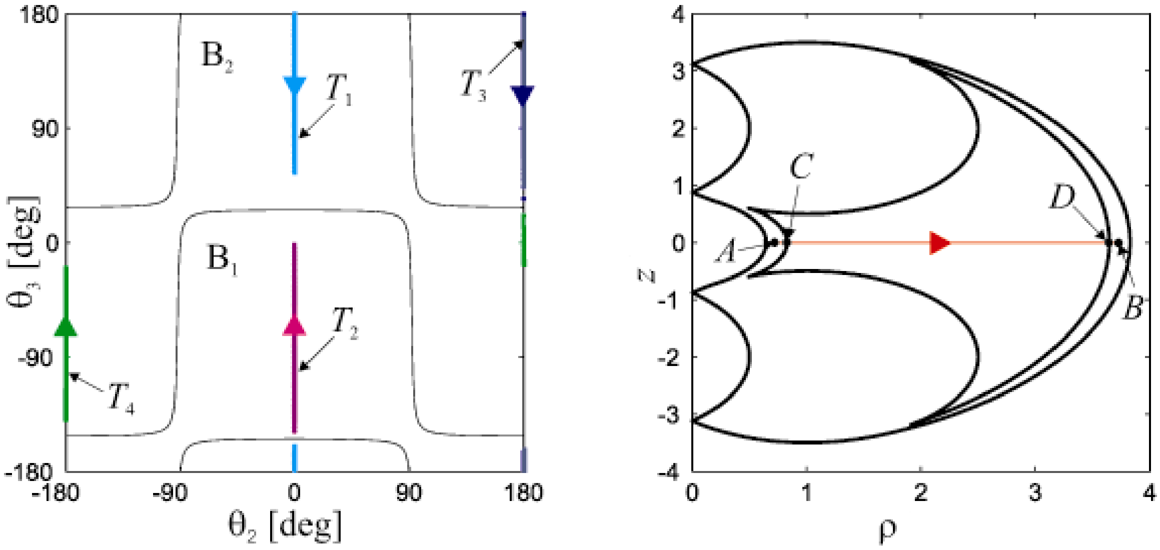


Figure 23: *AB* is not Path Feasible

Figure 23 depicts the joint space and the workspace of this manipulator. We can notice that it is generic because its singularity surfaces do not intersect in the joint space.

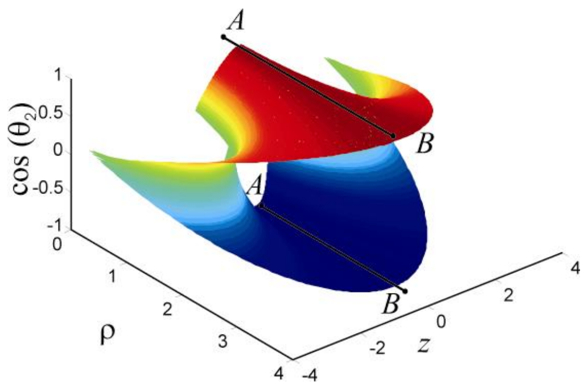


Figure 24: Image of aspect B_1: Region 1 of Feasible Paths

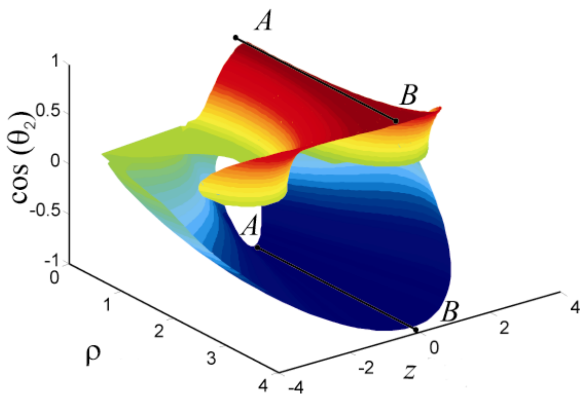


Figure 25: Image of aspect B_2: Region 2 of Feasible Paths

Moreover, it is cuspidal (it has four cusp points) and its joint space is composed of two aspects, B_1 and B_2. Their pre-images are depicted in Figs. 24 and 25, respectively. These figures show the regions of feasible paths of the manipulator too. None of these regions include path *AB*. Therefore, *AB* is not feasible by this generic manipulator.

However, we can notice that the main part of *AB* is feasible. Indeed, as depicted in Fig 23, the images of T_1, T_2, T_3, and T_4, defined in the joint space, are the segment lines *CB*, *CB*, *AD*, and *AD*, respectively.

Consequently, the feasibility of *AB* is very sensitive to variations in d_2 and the non-generic manipulator studied is not robust with respect to path feasibility.

In this paper, we do not claim that all generic manipulators are robust with respect to path feasibility. However, we noticed through some examples that the feasibility of a path can be very sensitive to the variations in geometric parameters when the manipulator is non-generic or close to a non-generic manipulator in the set of geometric parameters. Consequently, we state that the proximity of a manipulator to non-generic frontiers may severely affect its robustness with respect to path-feasibility.

C. Robustness with respect to the accuracy of the end-effector

Some properties of 3R non-generic manipulators and adjoined generic manipulators can be very sensitive to variations in geometric parameters, as explained in section III-B.

However, does it mean that these manipulators are less accurate? In order to answer this question, we compared the accuracy of many pairs of adjoined generic/non-generic manipulators.

Let us assume that the dimensional tolerances of the geometric parameters are known and are identical from one manipulator to the other: $\Delta d_2 = \Delta d_3 = \Delta d_4 = \Delta r_2 = \Delta r_3 = 0.1$ mm, $\Delta\alpha_2 = \Delta\alpha_3 = 5.10^{-4}$ rad, $\Delta\theta_2 = \Delta\theta_3 = 3.10^{-4}$ rad. Then, we can compute the maximum positioning error of the end-effector of the manipulator in its workspace.

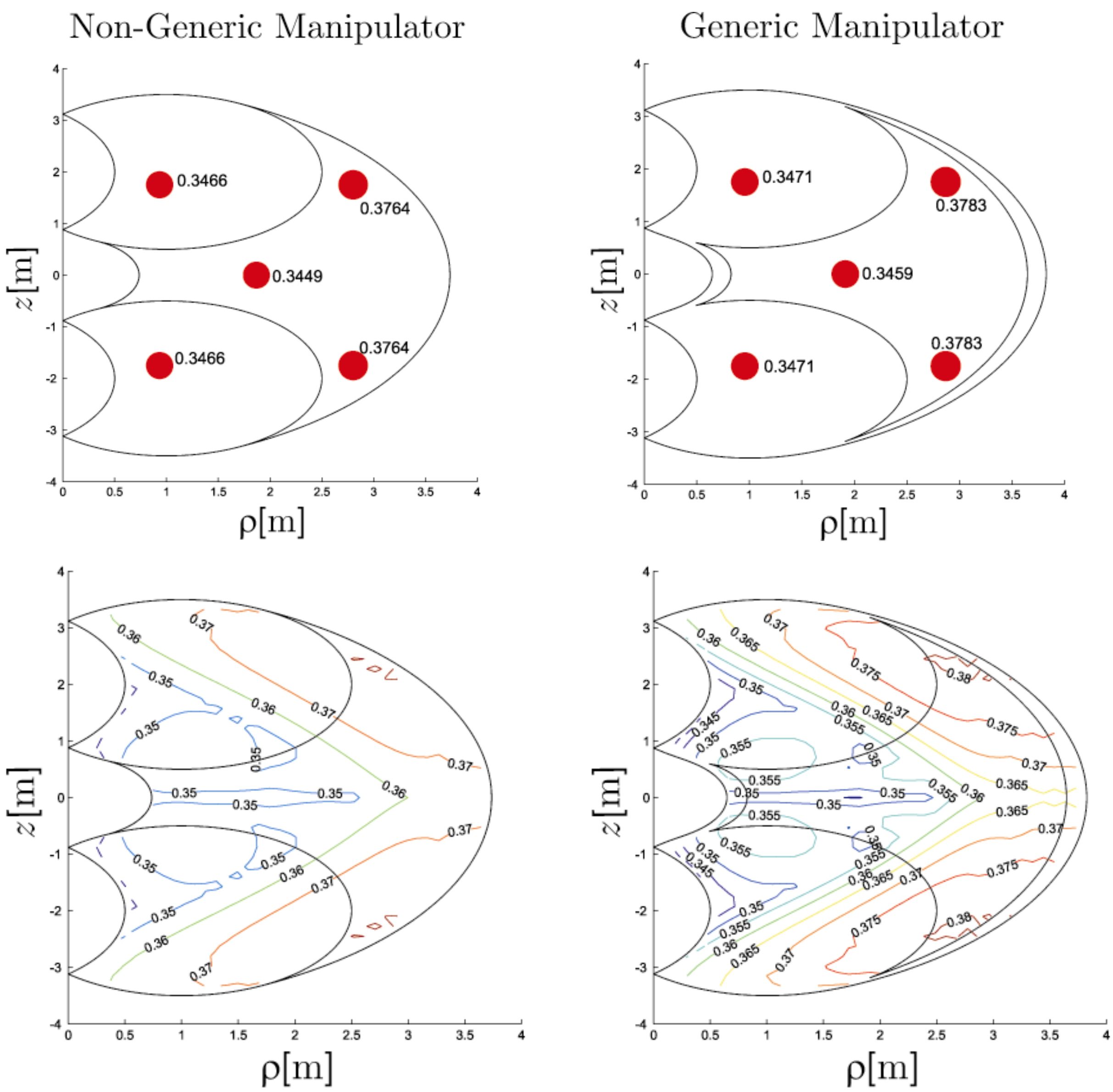


Figure 26: Maximum Positioning Error of the End-Effector of the Non-Generic (resp. Generic) Manipulator defined by $d_2 = 0$, (resp. $d_2 = 0.1$), $d_3 = 2$, $d_4 = 1.5$, $r_2 = 1$, $r_3 = 0$, $\alpha_2 = -90°$, $\alpha_3 = 90°$

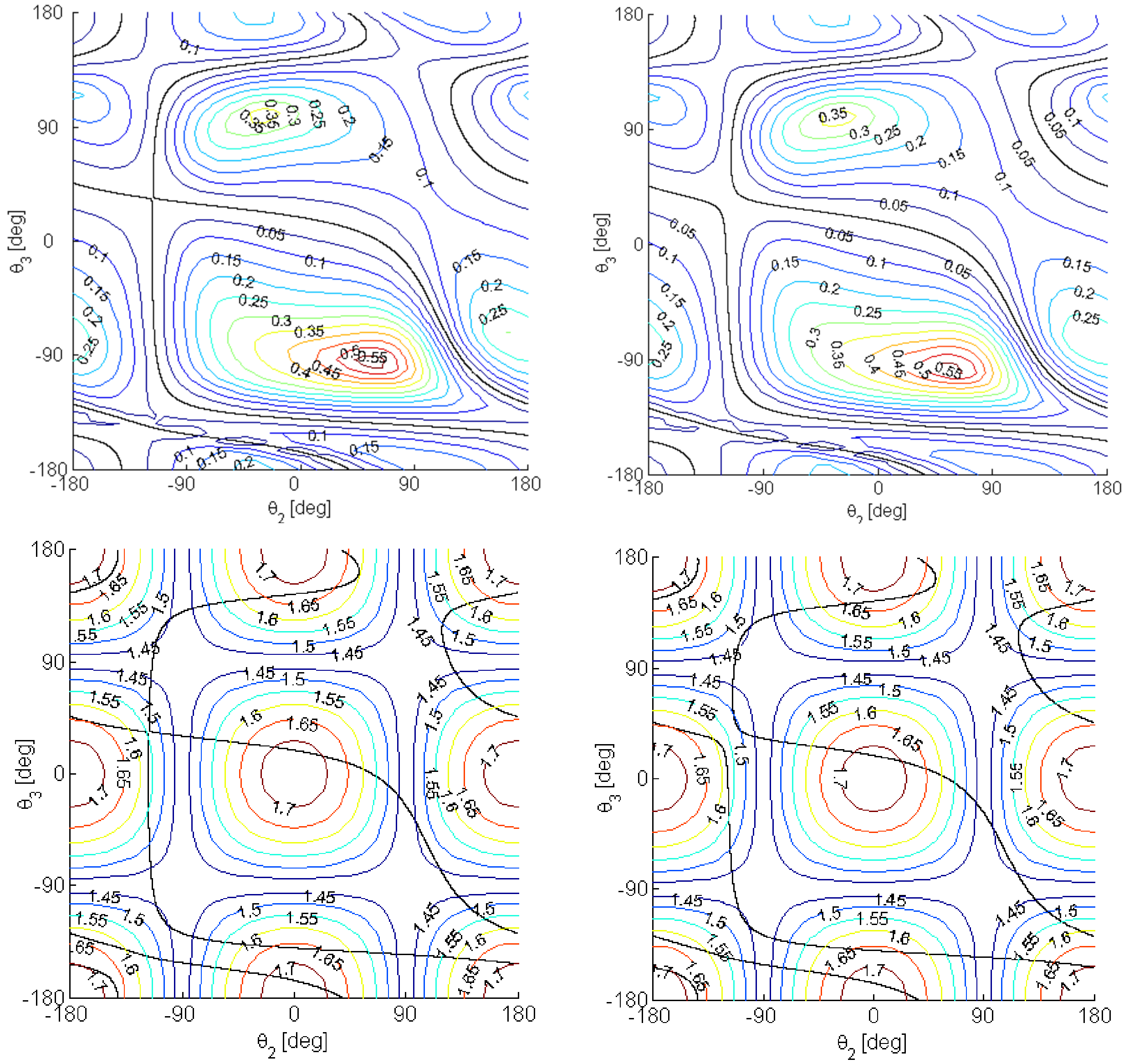
The first row of Fig. 26 shows the maximum positioning error of the end-effector of two adjoined non-generic and generic 3R manipulators (only d_2 changes from the non-generic manipulator to the generic one) computed at five points of their workspace. The second row depicts the iso-contours of the maximum positioning error of the end-effector of the manipulators. We plotted these graphs for many pairs of adjoined generic/non-generic manipulators. For each pair, we noticed that there are very few differences between the column related to the non-generic manipulator and the one related to the generic manipulator. Therefore, even if the comparative study is not general, we may conclude that the accuracy of the manipulator does not depend on the fact that the manipulator is generic or not.

IV. Kinetostatic Performances and Sensitivity Analysis

This section intends to the following question: do the kinetostatic performances of a manipulator depend on its genericity?

In order to answer this question, we applied an empirical approach. According to [17], the condition number of the Jacobian matrix of a manipulator can be used to quantify its dexterity, and then its kinetostatic performances.

Non-Generic Manipulator **Generic Manipulator**



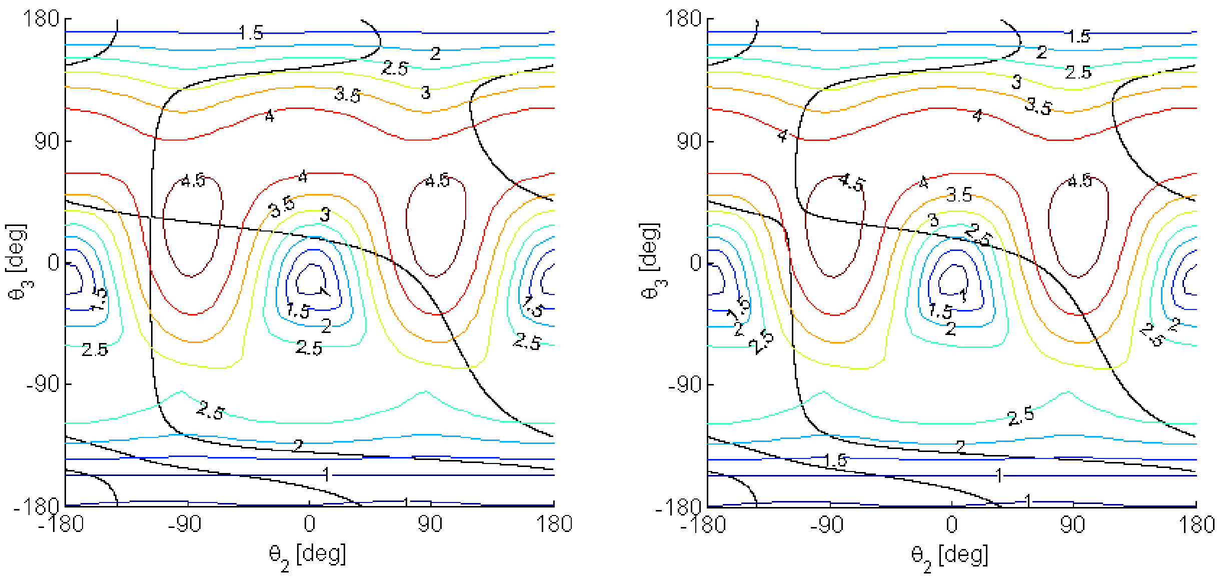


Figure 27: Kinematic Performances and Sensitivity Analysis of the Non-Generic (resp. Generic) Manipulator defined by: d_2 = 1, d_3 = 2, d_4 = 2.5, r_2 = 1, r_3 = 0.2, α_2 = -60° (resp. α_2 = -58°), α_3 = 90°

For instance, the first row of Fig.27 depicts the iso-contours of the inverse condition number of the Jacobian matrix of two non-generic and generic 3R manipulators close to each other in the set of geometric parameters. They are plotted in the joint space of the manipulators and are identical. Actually, they are the same for all the pairs of adjoined generic/non-generic manipulators that we studied.

Moreover, we analyzed and compared the sensitivity to length and angular variations of these manipulators. In order to evaluate this sensitivity, we used the optimal robustness index presented in [4], i.e., the 2-norm of the sensitivity Jacobian matrix of the manipulators, which maps the set of variations in the geometric parameters of the manipulators into the set of variations in their performances. Consequently, the second row of Fig.27 shows the iso-contours of the 2-norm of the sensitivity Jacobian matrix to length variations, of two adjoined non-generic and generic 3R manipulators, respectively. Likewise, the third row of Fig.27 shows the iso-contours of the 2-norm of the sensitivity Jacobian matrix to angular variations of the corresponding manipulators.

We can notice that the plots corresponding to the non-generic and generic manipulators are similar. As a matter of fact, it occurs with all the pairs of adjoined generic/non-generic manipulators that we studied. It means that the sensitivity of a manipulator to its length and angular variations does not depend on its genericity.

Furthermore, we noticed that the joint configurations corresponding to a good accuracy of the manipulators do not necessarily tally with the ones corresponding to a good dexterity, *i.e.*: the ones corresponding to a low condition number of the Jacobian matrix.

In conclusion, we showed by means of many examples that kinematic performances and sensitivity of 3R manipulators to geometric variations do not depend on the genericity. We chose some pairs of adjoined generic/non-generic manipulators representative of the population of 3R manipulators. Accordingly, we may assume that the foregoing comments are true for all the 3R manipulators.

V. Conclusion

In this paper, a robustness study of 3R manipulators was presented in order to know whether generic manipulators are more robust than their non-generic companions. Firstly, some properties specific to 3R manipulators were introduced, such as singularities, cuspidality, genericity, homotopy classes, and path feasibility.

It turns out that the farther a manipulator is from non-generic ones, the more robust it is with respect to homotopy class and number of cusps. Moreover, we noticed through some examples that the feasibility of a path can be very sensitive to the variations in geometric parameters when the manipulator is non-generic or close to a non-generic manipulator in the set of geometric parameters. Consequently, we state that the proximity of a manipulator to non-generic frontiers may severely affect its robustness with respect to path-feasibility.

Besides, we noticed through several examples that the accuracy and dexterity of generic and non-generic manipulators are similar. Finally, we pointed out that the joint configurations corresponding to a good accuracy of the manipulators do not necessarily tally with those corresponding to a good dexterity.

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SUMMARY

Robustness Study of Generic and Non-Generic 3R Positioning Manipulators

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Abstract— This paper presents a robustness study of 3R manipulators and aims at answering the following question: are generic manipulators more robust than their non-generic counterparts? We exploit several properties specific to 3R manipulators such as singularities, cuspidality, homotopy classes, and path feasibility, in order to find some correlations between genericity and robustness concepts. It turns out that the farther a manipulator is from non-generic ones, the more robust it is with respect to homotopy class and number of cusps. Besides, we state that the proximity of a manipulator to non-generic frontiers may severely affect its robustness with respect to path-feasibility. Finally, we notice that the dexterity and the accuracy of 3R manipulators do not depend on genericity.

Keywords—Robust design, genericity, path-feasibility, singularity, cuspidality, homotopy class, serial manipulator.